



Ice Sheet System model Mesh Generation

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Mesh generation in ISSM

- Mesh generation is crucial for ice sheet modeling
 - controls the space of solutions
 - Finer mesh more precise but more computationally intensive
 - ISSM has 3 main meshers:
 - ① squaremesh (for ISMIP tests)
 - ② triangle (from J. Shewchuk)
 - ③ bamg (adapted from F. Hecht)

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Usage

squaremesh generates structured uniform meshes for rectangular domain

→ needed for ISMIP tests

```
1 md=squaremesh(model,100,200,15,25);
```

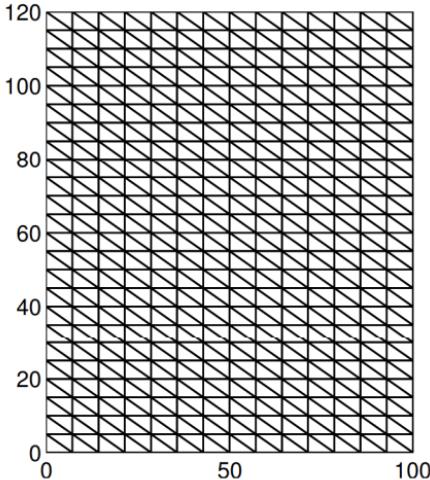
Arguments:

- ① model
- ② x-length
- ③ y-length
- ④ number of nodes along the x axis
- ⑤ number of nodes along the y axis

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Example

```
1 md=squaremesh(model,100,200,15,25);
```



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Usage

`triangle` is a very fast algorithm for mesh generation

- + excellent for uniform mesh
- bad at mesh refinement

```
1 md=triangle(model,'Square.exp',.2);
```

Arguments:

- ① model
- ② ARGUS file of the domain outline

```
## Name:domainoutline
## Icon:0
# Points Count  Value
5 1.
# X pos Y pos
0 0
1 0
1 1
0 1
0 0
```

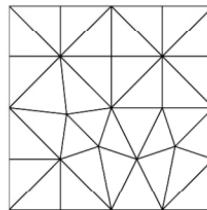
- ③ average element size



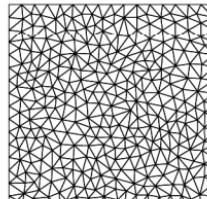
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Example

```
1 md=triangle(model,'Square.exp',.2);
```



```
1 md=triangle(model,'Square.exp',.05);
```

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History

Initial software:

- BAMG: Bidimensional Anisotropic Mesh Generator
- developed by Frédéric Hecht (INRIA/université de Jussieu)
- released in 2006 after more than 10 years of development
- now part of FreeFEM++

In ISSM:

- almost entirely rewritten
- usual ISSM interface

Advantages:

- + anisotropic mesh adaptation capability
- not good for uniform meshes

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Usage

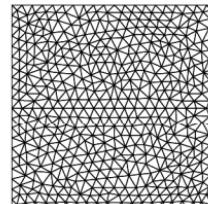
Arguments:

- ① model
- ② pair of options (see help)

To create a uniform mesh:

- ① 'domain' followed by the domain name
- ② 'hmax' followed by the triangle size

```
1 md=bamg(model,'domain','Square.exp','hmax',.05);
```



- Not as randomly distributed as triangle

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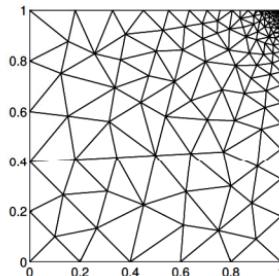
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Usage

To create a non-uniform mesh:

- ① 'domain' followed by the domain name
- ② 'hvertices' followed by the element size for each vertex of the domain outline

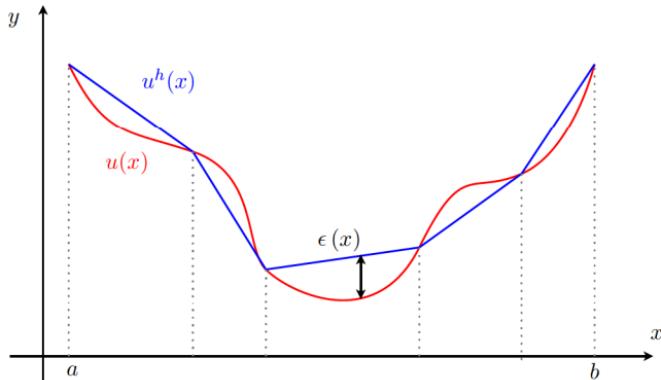
```
1 hvertices=[0.2;0.2;0.005;0.2];
2 md=bamg(model,'domain','Square.exp','hVertices',hvertices);
```

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Interpolation error

- We generally use piecewise linear elements ($P1$)
- How to minimize interpolation error and the number of elements at the same time?



Mesh generation

Larour et al.

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Example

Usage

Example

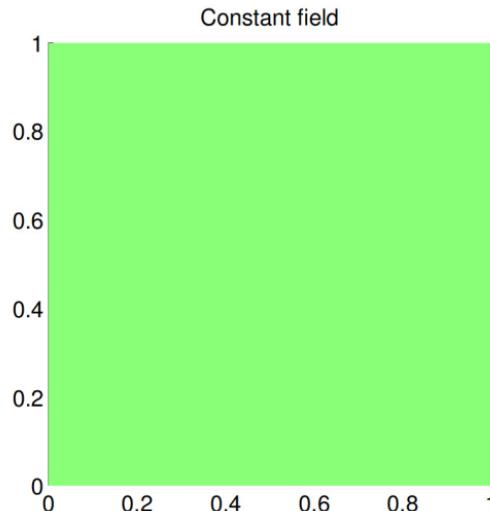
Uniform mesh

Non-uniform mesh

Mesh refinement strategy

Hands-on example

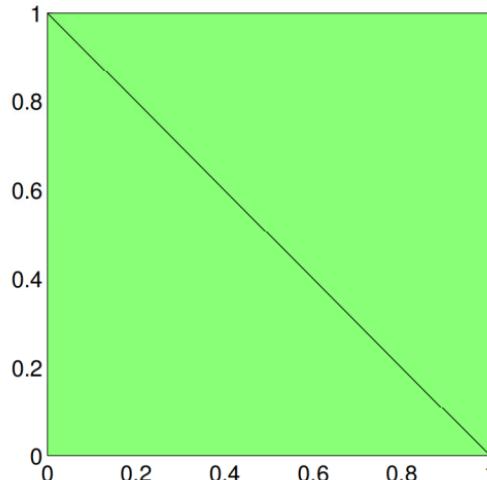
Costant field



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Costant field

Constant field

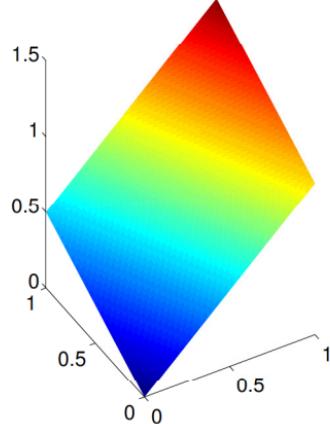


- Coarse elements OK for constant field

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Linear field

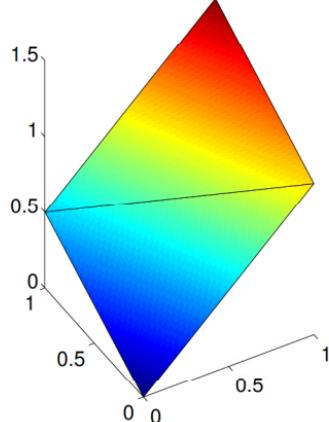
Linear field



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Linear field

Linear field



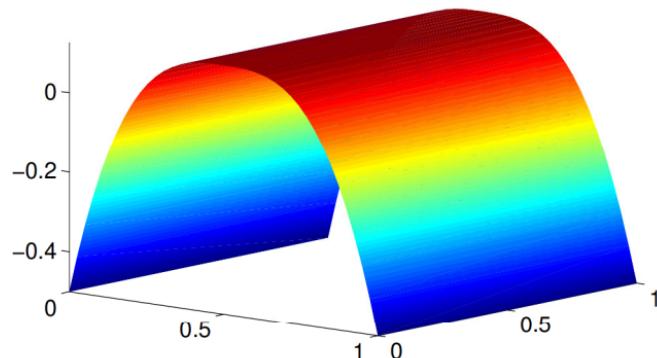
- Coarse elements OK for linear field

JPL

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Non-Linear field

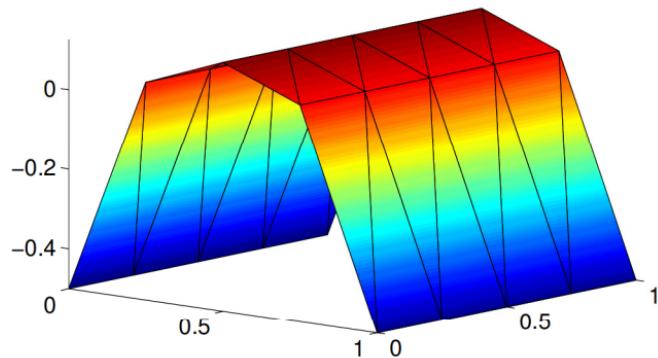
Non linear field



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Non-Linear field

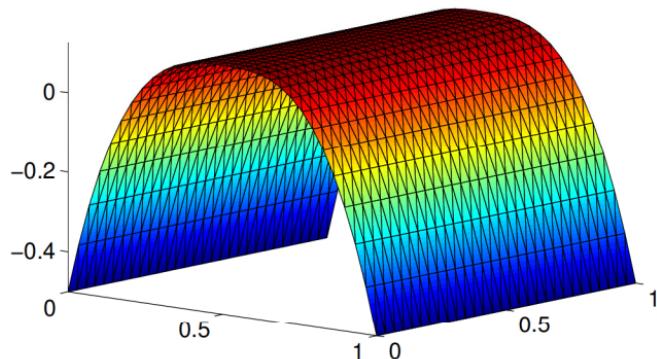
25 elements



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Non-Linear field

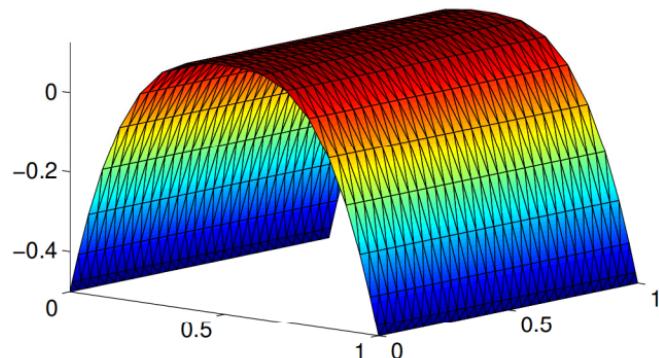
1,500 elements



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Non-Linear field

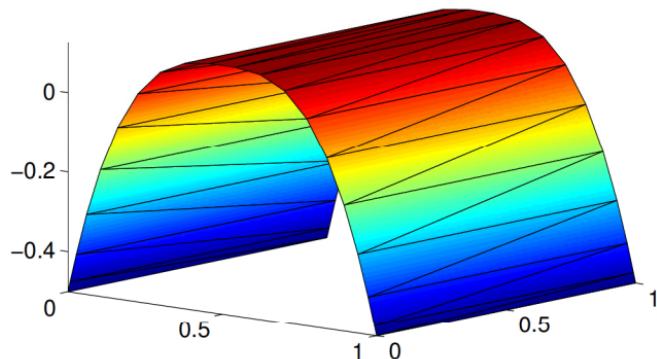
1,000 elements



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Non-Linear field

40 elements



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Anisotropic mesh refinement

Strategy:

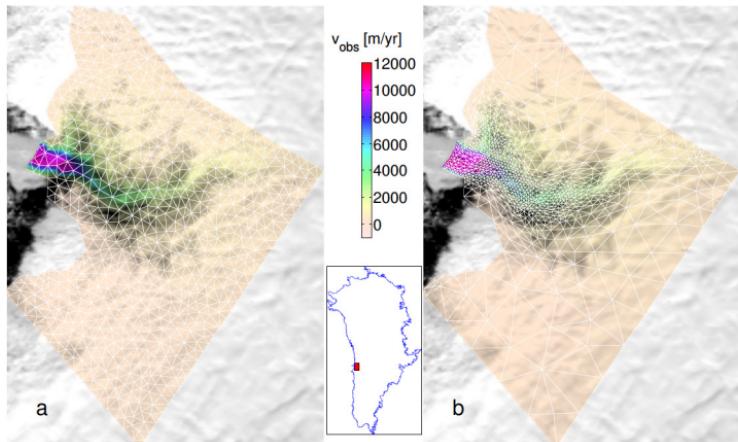
- Minimize the interpolation error for a given field
- Metric based on field's Hessian matrix (second derivative)

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Anisotropic mesh refinement

Strategy:

- Minimize the interpolation error for a given field
- Metric based on field's Hessian matrix (second derivative)

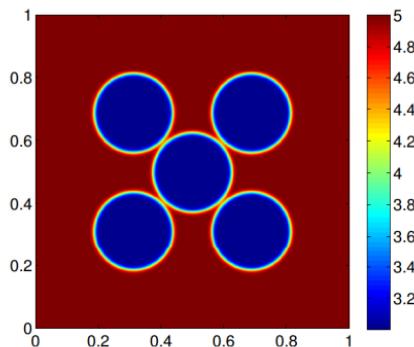


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Field to capture

$$\begin{aligned} f(x, y) = & \tanh(30(u^2 + v^2 - \varepsilon)) \\ & + \tanh(30((u - 0.75)^2 + (v - 0.75)^2 - \varepsilon)) + \tanh(30((u - 0.75)^2 + (v - 0.75)^2 + \varepsilon)) \\ & + \tanh(30((u - 0.75)^2 - (v - 0.75)^2 + \varepsilon)) + \tanh(30((u - 0.75)^2 - (v - 0.75)^2 - \varepsilon)) \end{aligned}$$

$$\text{with } \varepsilon = 0.25 \quad \text{and} \quad u = 4x - 2, \quad v = 4y - 2$$



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Uniform mesh

Non-uniform mesh

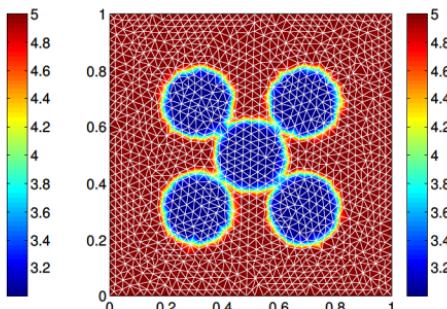
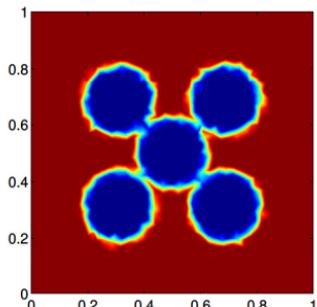
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Mesh refinement strategy

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Uniform mesh

```
1 md=bamg (model,'domain','Square.exp','hmax',.03);
2 vel=circles (md.mesh.x,md.mesh.y);
3 plotmodel(md,'data',vel,'edgecolor','w');
```



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Bamg

Uniform mesh

Non-uniform mesh

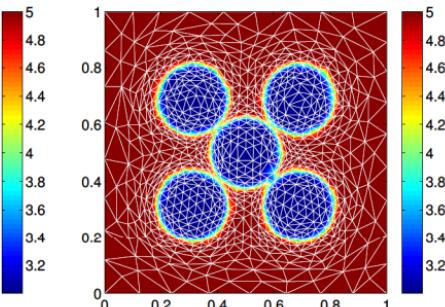
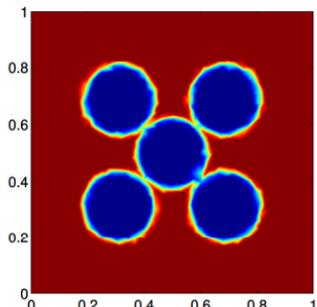
Mesh adaptation

Mesh refinement strategy

Hands-on example

Mesh refinement

```
1 md=bamg (model,'domain','Square.exp','hmax',.03);
2 vel=circles (md.mesh.x,md.mesh.y);
3 md=bamg (md,'field',vel,'err',0.05,'hmin',0.005,'hmax',0.3);
4 vel=circles (md.mesh.x,md.mesh.y);
5 plotmodel (md,'data',vel,'edgecolor','w');
```



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Uniform mesh

Non-uniform mesh

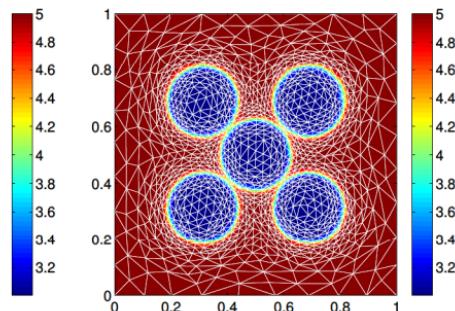
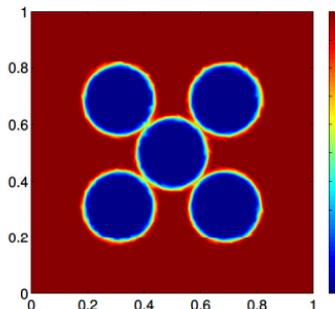
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Mesh refinement

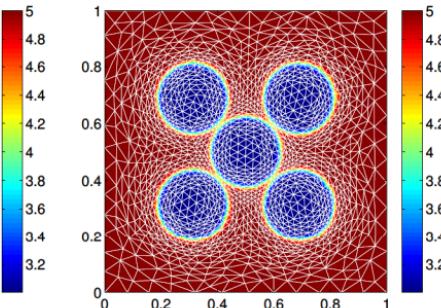
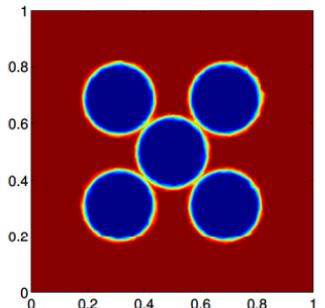
```
1 md=bamg (model,'domain','Square.exp','hmax',.03);
2 vel=circles (md.mesh.x,md.mesh.y);
3 md=bamg (md,'field',vel,'err',0.03,'hmin',0.005,'hmax',0.3);
4 vel=circles (md.mesh.x,md.mesh.y);
5 plotmodel (md,'data',vel,'edgecolor','w');
```



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Mesh refinement

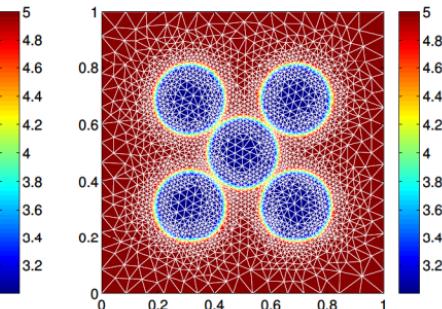
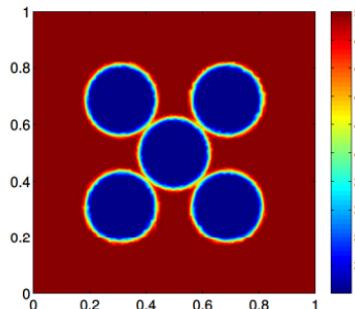
```
1 md=bamg(model,'domain','Square.exp','hmax',.03);
2 vel=circles(md.mesh.x,md.mesh.y);
3 md=bamg(md,'field',vel,'err',0.03,'hmin',0.005,'hmax',0.3,'gradation',1.3);
4 vel=circles(md.mesh.x,md.mesh.y);
5 plotmodel(md,'data',vel,'edgecolor','w');
```



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Mesh refinement

```
1 md=bamg(model,'domain','Square.exp','hmax',.03);
2 vel=circles(md.mesh.x,md.mesh.y);
3 md=bamg(md,'field',vel,'err',0.03,'hmin',0.005,'hmax',0.3,'gradation',1.3,'anisomax',1);
4 vel=circles(md.mesh.x,md.mesh.y);
5 plotmodel(md,'data',vel,'edgecolor','w');
```



A wide-angle photograph of a desolate, cold landscape. In the foreground, a flat expanse of white, textured snow or ice stretches across the frame. In the middle ground, a range of majestic, snow-capped mountains rises against a clear blue sky. The peaks are rugged, with deep shadows and bright highlights from the sunlight. The overall scene conveys a sense of vastness and silence.

Thanks!